Commun Math Phys 101, 213–245 (1985)

Communications in Mathematical Physics

Ground States of the XY-Model

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Abstract. Ground states of the XY-model on infinite one-dimensional lattice, specified by the Hamiltonian

$$-J[\sum\{(1+\gamma)\sigma_x^{(j)}\sigma_x^{(j)}+(1-\gamma)\sigma_y^{(j)}\sigma_y^{(j+1)}\}+2\lambda\sum\sigma_z^{(j)}]$$

with real parameters $J \neq 0$, γ and λ, are all determined. The model has a unique ground state for $|\lambda| \geqq 1$, as well as for $\gamma = 0$, $|\lambda| < 1$; it has two pure ground states (with a broken symmetry relative to the 180° rotation of all spins around the z-axis) for $|\lambda| < 1$, $\gamma \neq 0$, except for the known Ising case of $\lambda = 0$, $|\gamma| = 1$, for which there are two additional irreducible representations (soliton sectors) with infinitely many vectors giving rise to ground states.

The ergodic property of ground states under the time evolution is proved for the uniqueness region of parameters, while it is shown to fail (even if the pure ground states are considered) in the case of non-uniqueness region of parameters.

1. Main Results

We study ground states of the XY-model in the external transverse field on one-dimensional lattice (infinitely extended in two directions). Physical observables of the model are Pauli spins

$$\sigma_\alpha^{(j)} \quad (\alpha = x, y, z)$$

on each lattice site $j \in \mathbb{Z}$ ($[\sigma_\alpha^{(j)}, \sigma_\beta^{(k)}] = 0$ for $j \neq k$), which generates a UHF algebra $\mathfrak{A}$. The local Hamiltonian for an interval $[a, b]$ $(a < b)$ is

$$H(a,b) = -J\left[\sum_{j=a}^{b-1}\left\{\{(1+\gamma)\sigma_x^{(j)}\sigma_x^{(j+1)}+(1-\gamma)\sigma_y^{(j)}\sigma_y^{(j+1)}\}+2\lambda\sum_{j=a}^{b}\sigma_z^{(j)}\right.\right], \quad (1.1)$$

where J, γ (asymmetry of x and y), λ ($-2J\lambda$ being the strength of the external field) are real parameters and we assume* $J > 0$.

* The sign of the first summation in $H(a, b)$ can be inverted by 180° rotation of σ spins around the z-axis at every other site (for example at all odd sites) and the sign of the last summation can be inverted by the 180° rotation of all σ-spins around the x-axis, for example. Therefore the case of $J < 0$ can be reduced to the case of $J > 0$ under consideration