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A Classical Solution of the Non-Linear Complex Grassmann σ-Model with Higher Derivatives

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Abstract. We construct a soliton solution of the non-linear complex Grassmann σ-model with higher derivatives, and show that this solution, as a continuous map, represents a generator of the K-group of a sphere.

Introduction

Non-linear σ-models such as the CP^N σ-model or complex Grassmann σ-model in two dimensions are interesting objects to study not only for physicists but also mathematicians. They have non-instanton solutions with finite action other than instanton solutions. Moreover, a discrete symmetry transformation has been constructed in their solution spaces. See, in detail, [5] and its references.

In three or more dimensions, the situation is different. With usual action form, it is well known that a classical solution with finite action, which we call a soliton, does not exist, by the scaling argument of Derrick's type. Therefore we must alter the action to obtain a soliton.

In this note we construct a new Lagrangian on R^{2m} and show that it has at least one non-trivial soliton solution. Moreover we show that this one represents a generator of the K-group $\tilde{K}(S^{2m})(=Z)$ of the sphere S^{2m}.

I. The Model

We define a configuration space H which we consider hereafter. For natural numbers m, N we set

$$G_{2N,N} \equiv \{A \in M(2N;C) \mid A^2 = A, A^+ = A, \operatorname{Tr} A = N\}, \tag{1}$$

$$H_{2m} \equiv \{P: R^{2m} \to G_{2N,N},\ C^\infty\text{-class}\}. \tag{2}$$

It is known that $G_{2N,N}$ is a Grassmann manifold and $G_{2N,N} \cong \mathrm{U}(2N)/\mathrm{U}(N) \times \mathrm{U}(N)$. We call an element P in (2) a projector.

For the space H_{2m} we define a new Lagrangian as follows

$$L(P) \equiv \tfrac{1}{2}\int d^{2m}X \operatorname{Tr}(\partial_{\mu_1} \cdots \partial_{\mu_m} P)^2, \tag{3}$$
$$\partial_{\mu_j} \equiv \partial/\partial x_{\mu_j} \quad (j = 1, \ldots, 2m).$$