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The Method of Stationary Phase for Oscillatory Integrals on Hilbert Spaces

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Abstract. We develop the method of stationary phase for the normalized-oscillatory integral on Hilbert space, giving Borel summable expansions. The developments that we obtain hold for more general situations than the ones of previous papers on the same subject.

0. Introduction

In this paper we develop the method of stationary phase for oscillatory integrals on Hilbert spaces. Let us briefly describe the type of integrals we consider.

Let $\mathscr{H}$ be a real separable Hilbert space with inner product $(\cdot,\cdot)$. $\mathscr{M}(\mathscr{H})$ denotes the set of all bounded complex measures on $\mathscr{H}$ and $\mathscr{F}(\mathscr{H})$ denotes the set of their Fourier transforms: if $f\in\mathscr{F}(\mathscr{H})$, then $f:\mathscr{H}\to\mathbb{C}$ and

$$f(x)=\int_{\mathscr{H}}\exp\{i(x,\alpha)\}d\mu(\alpha) \tag{0.1}$$

for some $\mu\in\mathscr{M}(\mathscr{H})$.

In the following B and B^{-1} are bounded symmetric operators on $\mathscr{H}$. We study integrals of the form

$$I(h)=\tilde{\int_{\mathscr{H}}}\exp\left\{\frac{i}{h}\phi(x)\right\}g(x)dx, \tag{0.2}$$

where the integral is the normalized-oscillatory integral on $\mathscr{H}$ as defined in ref. [1], $\phi(x)$ is the real function $(1/2)\,(x,Bx)-V(x)$ and $V,\ g\in\mathscr{F}(\mathscr{H})$.

Integrals of this form arise in the theory of Feynman path integrals; see e.g. refs. [1–7]. In this case the elements of $\mathscr{H}$ are the paths of finite kinetic energy, $\phi(x)$ is the action along the path x, g is determined by the initial condition and h is Planck's constant divided by 2π. The method of stationary phase for (0.2) we shall develop, studying its asymptotic behaviour as $h\to 0$, is therefore connected with the heuristic ideas on the relation between quantum and classical mechanics.

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