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On the Hyperbolicity of Einstein's and Other Gauge Field Equations*

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Abstract. It is shown that Einstein's vacuum field equations (respectively the conformal vacuum field equations) in a frame formalism imply a symmetric hyperbolic system of "reduced" propagation equations for any choice of coordinate system and frame field (and conformal factor). Certain freely specifiable "gauge source" functions occurring in the reduced equations reflect the choice of gauge. Together with the initial data they determine the gauge uniquely. Their choice does not affect the isometry class (conformal class) of a solution of an initial value problem. By the same method symmetric hyperbolic propagation equations are obtained from other gauge field equations, irrespective of the gauge. Using the concept of source functions one finds that Einstein's field equation, considered as second order equations for the metric coefficients, are of wave equation type in any coordinate system.

1. Introduction

In this article rather general results on the hyperbolicity of Einstein's and other gauge field equations will be discussed. They were obtained by an analysis of the conformal vacuum field equations in the context of a specific initial value problem. The "conformal structure" of Einstein's vacuum field equations allows one to represent the conformal vacuum field equations

$$\mathrm{Ric}(\Omega^{-2}g) = 0 \tag{1.1}$$

for the "non-physical" metric g and the "conformal factor" Ω by a system of first order partial differential equations which remains regular even where Ω vanishes [1]. By this property of the field equations it is possible to transform global "initial value" problems for Einstein's vacuum field equations, where the solutions are to be characterized by their limiting behaviour near past null infinity, into local initial

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