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Topological Quantization and Cohomology*

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Abstract. The relationships between topological charge quantization, Lagrangians and various cohomology theories are studied. A very general criterion for charge quantization is developed and applied to various physical models. The relationship between cohomology and homotopy is discussed.

1. Introduction

Topology and geometry have been playing an important role in our current theoretical understanding of quantum field theories. One of the most interesting applications of topology has been the quantization of certain coupling constants. In this article we present a very general framework under which one can understand coupling constant quantization. Firstly, to argue that the correct framework for analyzing topological quantization is the mathematical discipline known as Cech cohomology. Secondly, to develop the ideas of Cech cohomology by using familiar examples from physics.

We will see that Dirac's magnetic charge quantization condition [1], the quantization of the coupling constant [2] in the Wess-Zumino Lagrangian [3], the quantization [4] of the mass term in three dimensional Yang-Mills theory [5], and the Bagger-Witten gravitational constant quantization [6] can all be formulated within the same framework.

The mathematical formalism we will discuss is standard mathematics [7] which is unfamiliar to most physicists. In this paper we stress the relationship of this abstract mathematical formalism to the Lagrangian mechanics of physicists. We will see that the origin of the quantization conditions is closely related to a famous theorem from classical mechanics that states that two Lagrangians which differ by a total derivative lead to equivalent dynamics. Generalizing some ideas of Wu and

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