Commun. Math. Phys. 100, 267–277 (1985)

Communications in
Mathematical
Physics

Global Aspects of Fixing the Gauge in the Polyakov String and Einstein Gravity

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Abstract. It is shown that there exists a possible obstruction to a continuous global gauge choice in the Polyakov string theory and in four dimensional Einstein gravity. In many circumstances this obstruction results in no global gauge existing in these two theories.

1. Introduction

The Feyman path integral approach to quantising gauge theories appears to be the best method available at present. It has been applied with considerable success to the quantisation of Yang-Mills theories and QCD. In the Euclidean path integral approach to Yang-Mills theories one considers functional integrals of the form

$$Z = \int_{\mathscr{C}} \mathscr{D}A \exp - S[A],$$

where $\mathscr{D}A$ is a measure on the space $\mathscr{C}$ of all gauge potentials A. $S[A]$ is the Yang-Mills action of A and the functional integral is taken over all gauge potentials which satisfy some suitable boundary condition. However, it is well known that there is a problem in evaluating this path integral which results from the gauge invariance of the action $S[A]$.

Let $\mathscr{G}$ denote the group of gauge transformations. The difficulty arises because the orbits of $\mathscr{G}$ are expected to have infinite measure. The functional integral should really be carried out over the gauge orbit space $\mathscr{C}/\mathscr{G}$. However, $\mathscr{C}/\mathscr{G}$ is an intractable space. The idea of fixing the gauge is intended to circumvent this difficulty. We choose, in a continuous way, one gauge potential on each $\mathscr{G}$-orbit, i.e., we choose a continuous map $s: \mathscr{C}/\mathscr{G} \to \mathscr{C}$ such that $p \circ s = \mathrm{id}$, where $p: \mathscr{C} \to \mathscr{C}/\mathscr{G}$ is the canonical projection. The functional integral is then evaluated over $s(\mathscr{C}/\mathscr{G})$, weighted by the Jacobian of $p: s(\mathscr{C}/\mathscr{G}) \to \mathscr{C}/\mathscr{G}$. This yields the Fadeev-Popov

* Work supported by the Science and Engineering Research Council
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