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On Non-Equilibrium Dynamics of Multidimensional Infinite Particle Systems in the Translation Invariant Case

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Abstract: It is shown that for a set of full measure with respect to any translation invariant probability distribution on the space of initial configurations of classical particle systems on $\mathbb{R}^d$ with interaction given by a smooth superstable potential of finite range there is a solution to the Newtonian equations of motion, provided that the specific energy and the particle density of the initial configuration exist a.s.

0. Introduction

The infinite particle system approach to statistical mechanics raises the question of the existence of Newtonian dynamics for *countable systems of interacting particles* in Euclidean space $\mathbb{R}^d$. This means the following (if we confine ourselves to the case of pairwise interaction): Let be given a *potential function* $U:\mathbb{R}^d \curvearrowright \mathbb{R}$ fulfilling some regularity properties including smoothness and decay conditions. We consider the following infinite system of ordinary differential equations, concerning a sequence $\{\mathscr{x}_i\}$, $i=1,2,\ldots$ of twice continuously differentiable functions $\mathscr{x}_i:[0,\infty)\curvearrowright\mathbb{R}^d$:

$$(N): \qquad \ddot{\mathscr{x}}_i = -\sum_{j\neq i} \nabla U(\mathscr{x}_i - \mathscr{x}_j), \qquad i=1,2,\ldots.$$

The problem is to prove for a largest possible class of initial conditions

$$\Phi_0 = \{[x_i, v_i]\}_{i=1,2\ldots}$$

[where $x_i = \mathscr{x}_i(0)$, $v_i = \dot{\mathscr{x}}(0)$] that a solution to (N) exists. Naturally, we suppose that the initial configuration is locally finite, i.e. $\{x_i\}_{i=1,2,\ldots}$ has no accumulation points.

This is a purely analytical question. It is well-known that in the case of dimension one and two for some classes of potential functions U Lanford [1] $(d=1)$, Dobrushin and Fritz [3,4] $(d=1,2)$, and Gurevich and Suhov [15] $(d=1)$ succeeded to show the existence and (in a certain sense) the uniqueness of a solution to (N) for large classes of initial configurations which are characterized by explicit conditions.