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Global Gravitational Anomalies

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Abstract. A general formula for global gauge and gravitational anomalies is derived. It is used to show that the anomaly free supergravity and superstring theories in ten dimensions are all free of global anomalies that might have ruined their consistency. However, it is shown that global anomalies lead to some restrictions on allowed compactifications of these theories. For example, in the case of O(32) superstring theory, it is shown that a global anomaly related to $\pi_7(O(32))$ leads to a Dirac-like quantization condition for the field strength of the antisymmetric tensor field.

Related to global anomalies is the question of the number of fermion zero modes in an instanton field. It is argued that the relevant gravitational instantons are exotic spheres. It is shown that the number of fermion zero modes in an instanton field is always even in ten dimensional supergravity.

I. Introduction

Anoamlies which arise in perturbation theory [1] can ruin the consistency of quantum field theories. Cancellation of such anomalies is a significant restriction on models in four dimensions [2], and even more so in higher dimensions [3], especially when gravitational and mixed gauge-gravitational anomalies are included [4]. Recently Green and Schwarz have exhibited a new mechanism for anomaly cancellation in higher dimensions [5], which leads to the existence of $n=1$ supersymmetric theories with "miraculous" anomaly cancellation in ten dimensions (and O(32) or $E_8 \times E_8$ gauge group). These theories are particularly attractive because they are the low energy limits of consistent superstring theories [6]. (The original discovery involved O(32), but a superstring theory based on $E_8 \times E_8$ has been discovered [7].) Such "miraculous" cancellation of anomalies also occurs [4] in the $n=2$ chiral supersymmetry theory in ten dimensions, but that

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