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Monopoles and Rational Maps: A Note on a Theorem of Donaldson

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Abstract. In a recent paper, Donaldson gave a description of the moduli space of SU(2) monopoles in terms of rational maps; this was done indirectly, via the associated solution of Nahm's equations. We give here an interpretation of these rational maps in terms of the monopole's spectral curve, and then as "scattering data" for the monopole itself.

1. Introduction

In [1], Donaldson, in proving a conjecture of Atiyah and Murray, gave a description of the moduli space M_k of SU(2)-monopoles (solutions to the SU(2) Bogomolny equations) of charge k, showing that the space R_k of rational maps $f: \mathbb{P}_1(C) \to \mathbb{P}_1(C)$ of degree k, such that $f(\infty) = 0$, could be identified with a circle bundle over M_k. The construction is done indirectly, using the fact [3, 4] that monopoles can be obtained, via an infinite dimensional ADHM construction, from solutions to the Nahm equations, i.e. $k \times k$ matrix valued functions $T_i(s)$, $i = 1, 2, 3$, $s \in (0, 2)$ such that

$$\frac{dT_i}{ds} + \tfrac{1}{2} \Sigma \varepsilon_{ijk} [T_j, T_k] = 0, \qquad (1)$$

$$T_i^*(s) = -T_i(s), \qquad (2)$$

$$T_i(2 - s) = T_i(s)^T, \qquad (3)$$

The T^i are analytic over (0, 2), with simple poles at 0, 2. (4)

The residues of T_i at $s = 0$ form an irreducible representation of su(2). (5)

To each $O(k, \mathbb{C})$ equivalence class of such solutions, Donaldson associates a circle of rational maps.

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