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Global Properties of Supermanifolds

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Abstract. We construct new examples of supermanifolds, and determine the vector bundle structure of the supermanifolds commonly used in physics. We show that any supermanifold admits a foliation whose leaves are locally tangent to the soul directions in the coordinate charts, and which is one of a nested sequence of foliations. We point out that the existence of these foliations implies restrictions on the possible topologies of supermanifolds. For example, a compact supermanifold with a single even dimension must have vanishing Euler characteristic. We also show that a globally defined superfield on a "nice" compact supermanifold must be constant along the leaves of the foliations. By this mechanism, the global topology of a supermanifold can be used to impose physically interesting constraints on superfields. As an example, we exhibit a supermanifold which has the local geometry of flat superspace but is such that all globally defined superfields are chiral.

1. Introduction

Superspace techniques have become standard tools for exploring the properties of supersymmetric field theories [1]. Various attitudes can be taken toward the anticommuting θ coordinates which are introduced in the superspace approach. One common attitude is to regard superspace as merely a formal or heuristic tool for obtaining results which could be more laboriously derived by other, more rigorous, methods. On this view, the anticommuting coordinates are merely formal symbols serving bookkeeping functions: keeping track of the particle content of supermultiplets as well as the differing commutation relations of bosonic and fermionic fields. Functions of the θ coordinates are understood in the sense of formal power series only. Although supersymmetry transformations can be formally represented as coordinate transformations in superspace, the real

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