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Effective Actions and Large-N Limits

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Abstract. The saddlepoint action for a large-N theory is given as an effective action for composite operators. This effective action is computed explicitly for $O(N)$ models and as a series in large-N invariants for matrix models. In the latter case, the use of the first term of the series is found to give good numerical agreement with the exact solutions of the solvable models.

Introduction

A very appealing approach to the solution of quantum field theories is the $\frac{1}{N}$ expansion [0]. One assumes that the number of fields is a power of a parameter N and that the coupling constants scale appropriately to give a large-N limit for correctly normalized expectations. It is then possible to expand the expectations as series in $\frac{1}{N}$. This nonperturbative approach to field theories gives rich physical information in the solvable examples. An interesting feature of the $N\to\infty$ limit is that it is given by extremizing a saddle point functional.

When the number of fields scales as N^1, this functional can be found by path integral methods. In the case of most physical interest, in which the number scales as N^2 (as in QCD), the $N\to\infty$ limit is unknown. We wish to explore this functional in both cases.

As an example of the N^1 case, consider the field space $\{\phi^i:\mathbb{R}^n\to\mathbb{R}^N\}$ and the (Minkowski space) Lagrangian

$$L(\phi^i)=\int\left[\frac{1}{2}(\partial\phi^i)^2-NV\left(\frac{1}{N}\phi^i\cdot\phi^i\right)\right]d^nX\,.$$

We wish to find the normalized free energy

$$Z=-i\frac{1}{N}\int\exp(i\mathscr{L}(\phi^i))\mathscr{D}\phi^i\,.$$