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# The Aetiology of Sigma Model Anomalies

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**Abstract.** Certain nonlinear sigma models with fermions are ill-defined due to an anomaly which exhibits characteristics of both the nonabelian gauge theory anomaly and the SU(2) anomaly. The simplest way to diagnose the anomaly involves consideration of the global topology of the theory. We review the mathematical methods needed for this analysis and apply them to several supersymmetric sigma models. Some of these are found to be anomalous.

## 1. Introduction

Quantum field theories of fermions interacting with nonabelian gauge fields sometimes exhibit an anomaly in the gauge current [1, 2]. This anomaly has recently attracted much attention [3–20], since it has become clear that it is usually a manifestation of a global obstruction to defining the theory properly (i.e. gauge invariantly).

A slight rephrasing of this result clarifies the main issue. Instead of formulating gauge theories in terms of the space $\mathscr{A}^{(4)}$ of connections on a principal bundle over Euclidean spacetime $X$, we can instead formulate them in terms of the space $\mathscr{C}^{(4)} \equiv \mathscr{A}^{(4)}/\mathscr{G}^{(4)}$ of gauge orbits in $\mathscr{A}^{(4)}$ [21, 22].[1] Now there is no question of gauge-dependence of the effective action. Instead the anomaly shows up as a topological obstruction to defining the dynamics of the fermion fields throughout $\mathscr{C}$ in a smooth, consistent way.

Unlike perturbation theory, which simply gives the gauge variation of the fermion effective action $\Gamma_f[A]$, the topological approach gives a direct geometrical interpretation of this variation. The situation is analogous to what we would have in general relativity were we to treat a tensor quantity, like energy density, as a scalar. Things might look acceptable as long as we worked in one coordinate frame. But if our manipulations required us to integrate this density over spacetime, we would be disappointed to find that the resulting number had no

1 We will henceforth drop the superscript '4' when no confusion can arise