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The Effect of Spherical Shells of Matter on the Schwarzschild Black Hole

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Abstract. Based on previous work we show how to join two Schwarzschild solutions, possibly with different masses, along null cylinders each representing a spherical shell of infalling or outgoing massless matter. One of the Schwarzschild masses can be zero, i.e. one region can be flat. The above procedure can be repeated to produce space-times with a C^0 metric describing several different (possibly flat) Schwarzschild regions separated by shells of matter. An exhaustive treatment of the ways of combining four such regions is given; the extension to many regions is then straightforward. Cases of special interest are: (1) the scattering of two spherical gravitational "shock waves" at the horizon of a Schwarzschild black hole, and (2) a configuration involving only *one* external universe, which may be relevant to quantization problems in general relativity. In the latter example, only an infinitesimal amount of matter is sufficient to remove the "Wheeler wormhole" to another universe.

1. Introduction

In a previous paper [1], we showed the existence of a spherical gravitational shock wave at the horizon of a Schwarzschild black hole due to a massless particle located there. We show here how to generalize this result to a spherical shell of matter which joins two Schwarzschild regions, possibly of different masses. We give an exhaustive treatment of the ways of joining four Schwarzschild regions (possibly flat). We then discuss several examples of special interest, including the scattering of *two* such shells of matter, and a model for a black hole with only *one* asymptotic region.

It is particularly important to notice that by gluing Schwarzschild solutions together this way one finds solutions of the Einstein field equations with energy-momentum $T_{\mu\nu}$ corresponding to a pressureless dust cloud moving with the speed of light at the boundary. Attention is given to the physical requirement that the energy density of this matter distribution be positive. The "cloud" then consists of ordinary massless particles moving parallel to each other. It is this result that

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