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Bunting Identity and Mazur Identity for Non-Linear Elliptic Systems Including the Black Hole Equilibrium Problem

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Abstract. Recent work by G. Bunting and by P. O. Mazur has developed new techniques for proving uniqueness theorems for extensive classes of non-linear elliptic boundary value problems including that of the equilibrium state of an electromagnetically charged black hole. These methods are described and compared. It is shown that the rather general class of harmonic mappings that can be dealt with by the Bunting method (which needs no internal symmetry group) can be regarded as a generalisation of the particular (totally symetric) class of non-linear σ-models that can be dealt with by the Mazur method.

Introduction and Background

The purpose of this work is to give a coherent description of the methods of proving uniqueness theorems for extensive classes of non-linear elliptic boundary value problems that have been developed recently by the independent work of Bunting [1, 2] and of Mazur [3]. It is shown that the particular class of non-linear σ-models (where the field lies in the quotient space of a non-compact Lie group by a maximal compact subgroup) to which the Mazur method applies can be considered as a special case of the more general class of harmonic (hypergeodesic) mappings (where the image space has negative curvature) to which the Bunting method applies.

The particular application that provided the original motivation for the development of these methods was a two-dimensional boundary problem for a well behaved but non-linearly self coupled elliptic system that was posed by the present author [4, 5] (see Appendix A) as the crux of a programme whose objective is to obtain a complete classification of all isolated black hole equilibrium states subject to the classical Einstein and Einstein-Maxwell field equations. The well known conclusion that for practical astrophysical purposes any isolated (asymptotically flat) pure vacuum black hole equilibrium state can be described by (a certain subclass of) the 2-parameter Kerr family of Einstein-solutions was effectively established by 1975 along lines originally sketched out by the present