Commun. Math. Phys. 99, 517–540 (1985)

Communications in Mathematical Physics

Monopole Charges for Arbitrary Compact Gauge Groups and Higgs Fields in any Representation

P. A. Horváthy[1] and J. H. Rawnsley[2]

1 Centre de Physique Théorique, CNRS, Luminy Case 907, F-13288 Marseille, Cedex 9, France
2 Mathematics Institute, University of Warwick, Coventry CV4 7AL, England

Abstract. The topological invariants of monopoles are described for an arbitrary compact gauge group G and Higgs field Φ in any representation. The results generalize those obtained recently for compact and simply connected G and Φ in the adjoint representation. The cases when the residual symmetry group is $H = \mathrm{U}(1)$ or $H = \mathrm{U}(3)$ are worked out explicitly. This latter is needed to accommodate fractional electric charge with monopoles having one Dirac unit magnetic charge.

The general theory is illustrated on the SU(5) monopole.

1. Introduction

Let us consider a gauge theory with a compact gauge group G and the Higgs field transforming according to an arbitrary representation of G. The coupled Yang–Mills–Higgs equations admit monopole solutions (see [14] for a review). Let us consider such a monopole given by the pair (A_j, Φ), and denote by H the residual symmetry group left by the vacuum expectation value of Φ.

In the theory of monopoles a fundamental role is played by the topological invariants [2–4, 8–10, 15, 21]. The most important of these invariants is

(i) The Higgs charge $[\Phi]\in\pi_2(G/H)$ defined by the asymptotic values of the Higgs field.

(ii) If Φ belongs to the adjoint representation, we have another topological invariant-the so-called topological charge

$$I = \int_{S^2} \mathrm{Tr}(F\cdot\Phi), \tag{1.1}$$

where F is the gauge field strength. Equation (1.1) appears for example in the expression given by Bogomolny to the lower bound of the energy. Equation (1.1) has been generalized by Taubes [2]. In [1] we made one further step and proved that, for any $(n+1)$-linear function f on $\mathscr{G}$ the integral

$$I^{(f)} = \int_{S^2} f(F, \underbrace{\Phi, \dots, \Phi}_{n \text{ times}}) \tag{1.2}$$