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The Coherent Potential Approximation is a Realizable Effective Medium Scheme

G. W. Milton*

Baker Laboratory, Cornell University, Ithaca, NY 14853, USA

Abstract. The effective electrical conductivity of an aggregate, composed of grains of various conductivities, is frequently estimated by the coherent potential approximation, which embodies a local effective medium concept. It is proved rigorously that this approximation is exact for a wide class of hierarchical model composites made of spherical grains: the starting material 0 in the hierarchy is chosen arbitrarily, otherwise, material $j=1,2,\ldots$ consists of equisized spheres, say j-spheres, of arbitrary conductivities embedded in material $j-1$. The spatial distribution of the j-spheres must satisfy a mild homogeneity condition and their radius r_j must, asymptotically, increase faster than exponentially with j. Furthermore, the minimum spacing, $2s_j$, between the j-spheres is such that the ratio s_j/r_j diverges. On the basis of these and some further ancillary conditions it is established that the coherent potential approximation becomes asymptotically exact for the effective conductivity of material $j\to\infty$. The results extend to other effective parameters of the composites, including the thermal conductivity, dielectric constant and magnetic permeability. In addition, the model composites and the proof of realizability may be generalized to allow non-spherical grains.

1. Introduction

Various theoretical approaches have been developed to deal with the classical problem of estimating the effective or average conductivity of a composite material given the geometry of the composite and the conductivities of the components: see for example, the reviews of Beran (1968), Batchelor (1974), Abeles (1976), Hale (1976), Landauer (1978), Bergman (1978), and McPhedran et al. (1983).

One approach is to solve the field equations directly with the aid of a computer; this has been successfully applied by McKenzie et al. (1978), Kantor and Bergman (1982), and Sangani and Acrivos (1983), among others, to periodic structures such

* Present address: California Institute for Technology, 405-47, Pasadena, CA 91125, USA