Commun. Math. Phys. 99, 451–460 (1985)

Communications in
Mathematical
Physics

Regularized and Renormalized Bethe-Salpeter Equations: Some Aspects of Irreducibility and Asymptotic Completeness in Renormalizable Theories

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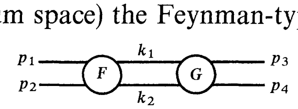
Abstract. Results on the links between 2-particle irreducibility and asymptotic completeness are presented in the framework of a renormalized Bethe-Salpeter formalism, introduced recently by J. Bros from an axiomatic viewpoint, for the most simple class of renormalizable theories. These results, which involve the *renormalized* 2-particle irreducible kernel G (i.e. from the perturbative viewpoint the sum of renormalized Feynman amplitudes of 2-particle irreducible graphs in the channel considered), complement the general quasi-equivalence previously established by Bros for *regularized* (non-renormalized) Bethe-Salpeter kernels. On the one hand, a formal derivation of (2-particle) asymptotic completeness from the irreducibility of G is given. On the other hand, the links between regularized and renormalized kernels are investigated. This analysis provides in particular a converse derivation (up to some assumptions) of the 2-particle irreducibility of G from asymptotic completeness. As a byproduct, it also provides a more explicit justification of previous heuristic derivations by K. Symanzik of integral equations between F and various differences of values of G, and a simple alternative derivation of the recently proposed "renormalized" Bethe-Salpeter equation.

1. Introduction

The usual Bethe-Salpeter equation reads:

$$F = G + F \circ G, \tag{1}$$

where F is the $2 \to 2$ Green function, G is the 2-particle irreducible (2 p.i.) Bethe-Salpeter kernel in the $2 \to 2$ channel considered, i.e. is from the perturbative viewpoint the sum of all Feynman amplitudes of 2-particle irreducible graphs, and $F \circ G$ denotes (in momentum space) the Feynman-type convolution integral



with Feynman propagators or two-point functions attached to each internal line; p_1, p_2 and p_3, p_4 denote the initial and final energy-momenta respectively and