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Nonlinear Stability of Circular Vortex Patches*

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Abstract. This paper proves that circular vortex patches in the plane are stable for the nonlinear dynamical system generated by the Euler equations of incompressible fluids. This is achieved by establishing a relative variational principle in terms of either energy or angular momentum. Thus, we exploit and extend Arnold's idea in (1965, 1969) to a nonsmooth setting as well.

1. Introduction

This paper proves that circular vortex patches in the plane (the vorticity is one inside a circle and is zero outside it) are stable for the nonlinear dynamical system generated by the two dimensional Euler equations of incompressible hydrodynamics. The stability is of Liapunov type: it is global in time in the L^1 norm on the vorticities. A consequence of the *a priori* estimates used to establish this stability is the following: a nearly circular vortex patch must evolve in such a way that the area of the region of deviation from circularity it uniformly bounded, globally in time. Our vortex patch is enclosed in a (large) circular disk, and the flow is parallel to the boundary of this containing disk.

Our results are stated precisely in Sect. 2. Here we comment on some of the relevant literature and the significance of the results. Kelvin (1880) showed that a small perturbation of the circular vortex patch which is proportional to cos $m\theta$ rotates uniformly with angular velocity $\Omega_m = \frac{1}{2}(m-1)/m$ (see Lamb, 1945, Sect. 158). In particular his result established the linearized stability of the circular patch in the plane.

In Arnold (1965), (1969), a method for proving a nonlinear version of the classical Rayleigh inflection point criterion for linearized stability of two dimensional shear flows is presented. The argument involves a combination of a geometric setting for the fluid variational principle and convexity arguments. The geometric setting has been exploited by a number of authors such as Ebin and Marsden (1970), Benjamin

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