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Massless Lattice ϕ_4^4 Theory: Rigorous Control of a Renormalizable Asymptotically Free Model

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Abstract. Using block spin renormalization group techniques, we rigorously control the functional integral of a weakly coupled critical lattice ϕ^4 theory in four euclidean dimensions proving the infrared asymptotic freedom of the model. This solves the infrared counterpart of and sheds some light on the problem of existence of continuum renormalizable ultraviolet asymptotically free models.

1. Introduction

One of the fundamental problems of Quantum Field Theory (QFT) is the existence of non-trivial models describing couplings of fields and scattering of particles. Such models do exist on the level of formal renormalized perturbation series, where renormalization removes the ultraviolet (UV) divergences of the naive perturbation expansion. The problem of non-perturbative existence can be viewed as equivalent to a non-perturbative understanding of renormalization. Up to now, the attempts at a non-perturbative control of the QFT models (constructive QFT [23], exactly soluble models [26]), although very instructive, have failed to produce quantum field theories in four space-time dimensions.

Much of our present understanding of the existence problem for QFT comes from the Renormalization Group (RG) approach. The RG, in its most full-fledged version [41] cast into the statistical mechanical framework in the euclidean space-time, replaces the static point of view of renormalized perturbation theory by a dynamical one. We try to see, mostly also perturbatively, how the local (euclidean) field theory may be obtained from its cut-off non-local versions in which the source of the troubles: the short distance (UV) singularities are regularized in order to guarantee the existence of the model. One of the crucial concepts arising from the RG approach is that of (UV) asymptotic freedom [25, 35]: a model is UV asymptotically free if its short distance asymptotics is non-interacting (free).

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