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New Compactifications of Ten and Eleven Dimensional Supergravity on Manifolds Which Are Not Direct Products

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Abstract. Solutions of ten and eleven dimensional supergravity are presented for which the space-time metric is a product of an anti-de Sitter metric and a strictly positive scalar function of the internal coordinates. The critical point of the potential of seven dimensional maximal supergravity with SO(4) symmetry is identified with such a solution.

In this letter we present new solutions of Kaluza-Klein supergravity for which the metric is of the form:

$$g_{\hat{\mu}\hat{\nu}}(x,y,\alpha)=\begin{pmatrix}\sigma^2(\alpha)g_{\mu\nu}(x) & 0 & 0\\ 0 & \varrho^2(\alpha)g_{\beta\gamma}(y) & 0\\ 0 & 0 & \tau^2(\alpha)\end{pmatrix}. \tag{1}$$

The tensor $g_{\mu\nu}(x)$ is the metric for any Einstein space-time with positive cosmological constant and dimension D_1, and $g_{\beta\gamma}(y)$ is any Einstein metric with negative cosmological constant, describing (D_2-1) dimensional internal space. The scaling functions ϱ and σ depend only upon the remaining internal coordinate, α. To set our conventions: indices, $\beta,\gamma,\delta,\ldots$ and the coordinates y^β refer to the (D_2-1)-dimensional internal space; $\mu,\nu,\varrho,\ldots$ and the coordinates x refer to space-time; $\hat{\mu},\hat{\nu},\hat{\varrho},\ldots$ will refer to the entire (D_1+D_2)-dimensional space-time, and the coordinates (x^μ, y^β, α) will be denoted generically by $z^{\hat{\mu}}$. The indices $a,b,c,\ldots$ and the coordinates w^a will refer to the entire D_2-dimensional internal space, parametrized by (y^β,α).

The metric (1) no longer describes the direct product of a D_1-dimensional space-time with a D_2-dimensional internal space. Indeed, the curvature of space-time depends on the internal space. However, it has already been shown [1], and we will demonstrate here, that such rescalings of the space-time metric are

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