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Statistical Mechanical Methods in Particle Structure Analysis of Lattice Field Theories

II. Scalar and Surface Models

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Abstract. We illustrate on simple examples a new method to analyze the particle structure of lattice field theories. We prove that the two-point function in Ising and rotator models has an Ornstein-Zernike correction at high temperature. We extend this to Ising models at low temperatures if the lattice dimension $d \geqq 3$. We prove that the energy-energy correlation function at high temperatures (for Ising or $N=2$ rotators) decays according to mean field theory (i.e. with the square of the Ornstein-Zernike correction) if $d \geqq 4$. We also study some surface models mimicking the strong-coupling expansion of the glueball correlation function. In the latter model, besides Ornstein-Zernike decay, we establish the presence of two nearly degenerate bound states.

I. Introduction

There has recently been some renewed interest in the analysis of the particle structure of (lattice) field theories, in particular of gauge theories [1–11]. In [11] we have outlined a new method leading to various results on the particle structure of scalar and gauge lattice field theories. In this paper, we explain our method in mathematical detail on the simplest examples and we prove some of the results claimed in [11].

We ask the following questions: what is the precise long-distance behaviour of the two-point function, or of higher-order correlation functions in lattice field theories? What information on the spectrum of the theory can one obtain from this behaviour? The connection between both questions is provided by the Källen-Lehman representation, which, for a continuum Euclidean theory, is:

$$\langle \phi_0 \phi_x \rangle = \int (-\Delta + a^2)^{-1}(0,x) d\varrho(a)\,, \tag{1.1}$$

where $d\varrho$ is a positive measure. An analogous, slightly weaker, formula holds for a reflection positive correlation function in a lattice theory. From (1.1) it is clear that, if we can prove that

$$\langle \phi_0 \phi_x \rangle \underset{|x| \to \infty}{\sim} \frac{e^{-m|x|}}{|x|^{(d-1)/2}} \sim (-\Delta + m^2)^{-1}(0,x) \tag{1.2}$$