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A Generalized Fluctuation-Dissipation Theorem for the One-Dimensional Diffusion Process

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Abstract. The $[\alpha, \beta, \gamma]$-Langevin equation describes the time evolution of a real stationary process with T-positivity (reflection positivity) originating in the axiomatic quantum field theory. For this $[\alpha, \beta, \gamma]$-Langevin equation a generalized fluctuation-dissipation theorem is proved. We shall obtain, as its application, a generalized fluctuation-dissipation theorem for the one-dimensional non-linear diffusion process, which presents one solution of Ryogo Kubo's problem in physics.

1. Introduction

In order to clarify a probabilistic meaning of the concept of T-positivity (reflection positivity) with its origin in axiomatic quantum field theory [3, 14], we have investigated a real stationary Gaussian process X having T-positivity from the viewpoint of the theory of stochastic differential equations [10, 11, 13]. In the previous paper [11], we characterized a class of stochastic differential equations describing the time evolution of X as a $[\alpha, \beta, \gamma]$-Langevin equation and then obtained a fluctuation-dissipation theorem for this $[\alpha, \beta, \gamma]$-Langevin equation as a generalized fluctuation-dissipation theorem in the theory of Ornstein-Uhlenbeck Brownian motion in statistical physics [2, 6–8, 15].

The purpose of the present paper is to refine the results of [11] and then make them serve to get a generalized second fluctuation-dissipation theorem for the one-dimensional non-linear diffusion process, which presents one solution of Kubo's problem in physics [6–8]. Before reformulating Kubo's problem stated in [7], we shall recall briefly a second fluctuation-dissipation theorem for Ornstein-Uhlenbeck Brownian motion. Let $\mathscr{X} = (\mathscr{X}(t), P_x; t \in [0, \infty), x \in \mathbb{R})$ be an Ornstein-Uhlenbeck Brownian motion whose time evolution is governed by the following stochastic differential equation:

$$\left.\begin{aligned} d\mathscr{X}(t) &= -\beta \mathscr{X}(t)dt + \alpha dB(t) \quad (t \in (0, \infty)) \\ \mathscr{X}(0) &= x . \end{aligned}\right\} \tag{1.1}$$

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