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Regularized Determinants for Quantum Field Theories with Fermions

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Abstract. A new type of regularized determinant for the ratio of two Dirac operators is presented. Some of its properties with application to the chiral anomaly are given.

1. Introduction

The Fredholm determinants have been a tool for formulating quantum field theories involving fermions since they appeared in the Matthews and Salam formulae which express the Green's functions of a Yukawa theory [1]. In order to avoid the divergences of the determinants in those formulae, some regularization procedure is needed [2–4].

Meanwhile, Fujikawa argued, in the formalism of quantum field theories using the integration on a Grassman algebra [5], about a certain kind of regularization necessary to get the chiral anomaly [6] correctly [7].

The purpose of this paper is to present a definition of a new type of regularized determinant for the ratio of two Dirac operators in which Fujikawa's idea is adopted. The intuitive idea of definition of our regularized determinant is the following. Suppose that we get an operator D_1 from another operator D_0 by performing successive infinitesimal transformations. Then the determinant of $D_1 D_0^{-1}$ is the product of the Jacobians of all the infinitesimal transformations. Our regularized determinant of $D_1 D_0^{-1}$ is obtained by replacing these Jacobians with their regularized ones introduced in [7, 8].

We explain this procedure more explicitly. Let D be a suitable operator-valued map defined on the interval $[0,1]$ which connects D_0 and D_1, i.e. $D(1)=D_1$, $D(0)=D_0$. If the operator $(dD(s)/ds)D(s)^{-1}$ is trace class for every $s\in I$, then we can get for the Fredholm determinant of $D_1 D_0^{-1}$ the following expression

$$\det D_1 D_0^{-1} = \exp\left(\mathrm{Tr} \int_0^1 \frac{dD(s)}{ds} D(s)^{-1}\, ds \right). \tag{1.1}$$