Commun. Math. Phys. 98, 253–258 (1985)

Communications in Mathematical Physics

Recurrence of Random Walks in the Ising Spins

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Abstract. Consider the 1/2-Ising model in Z^2. Let σ_j be the spin at the site $(j, 0) \in Z^2$ $(j=0, \pm 1, \pm 2, \ldots)$. Let $\{X_n\}_{n=0}^{+\infty}$ be a random walk with the random transition probabilities such that

$$P(X_{n+1}=j\pm 1|X_n=j)=p_j^{\pm}\equiv 1/2\pm v(\sigma_j-\mu)/2.$$

We show a case where $E[p_j^+] \gtreqless E[p_j^-]$, but $\lim_{n\to\infty} X_n = -\infty$ a.s. or X_n is recurrent a.s.

Let $\{\sigma_j\}_{j=-\infty}^{+\infty}$ be an ergodic random sequence of ± 1 spins with the mean $E[\sigma_j]=m$. Considering $-\sigma_j$ if $m<0$, we may assume $0 \leqq m < 1$. Let $\{X_n\}_{n=0}^{+\infty}$ be a random walk with random transition probabilities such that

$$P(X_{n+1}=j+1|X_n=j)=p_j^+ \equiv 1/2+v(\sigma_j-\mu)/2,$$
$$P(X_{n+1}=j-1|X_n=j)=p_j^- \equiv 1/2-v(\sigma_j-\mu)/2,$$

where v and μ are constants with

$$|v|(1+|\mu|)<1.$$

We are interested in the recurrence of the random walk $\{X_n\}_{n=0}^{+\infty}$. Since the recurrence is trivial if $v=0$, let us assume $v \neq 0$. We apply Chung's results, which are summarized in the following

Lemma 1 (Sect. 12, Part I in [1]). *Let $\{X_n\}_{n=0}^{+\infty}$ be a random walk with non-random positive transition probabilities $p_j^{\pm}$ $(p_j^+ + p_j^- = 1)$ which depend on j, i.e.,*

$$P(X_{n+1}=j\pm 1|X_n=j)=p_j^{\pm}.$$

i) *If* $\sum_{r=1}^{+\infty} p_1^- p_2^- \ldots p_r^-/(p_1^+ p_2^+ \ldots p_r^+) = \sum_{r=-\infty}^{0} p_r^+ p_{r+1}^+ \ldots p_0^+/(p_r^- p_{r+1}^- \ldots p_0^-) = +\infty$, *then $\{X_n\}_{n=0}^{+\infty}$ is recurrent a.s.*