Commun. Math. Phys. 98, 245–251 (1985)

Communications in
Mathematical
Physics

Borel-Le Roy Summability of the High Temperature Expansion for Classical Continuous Systems

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Abstract. For classical gases with suitable pair interactions such that $\Phi(r)\sim(\ln r^{-1})^p$ as $r\to 0$ $(p\in\mathbb{N})$, the Taylor expansion in β of the correlation functions and the pressure are summable at $\beta=0$ by the Borel-Le Roy method of order $p+1$.

I. Introduction

As it is known [5], for classical continuous systems with stable and regular pair potentials the correlation functions and the pressure admit a convergent power series expansion in the activity z, while the typical analyticity region in β $(\beta=(kT)^{-1})$ is the half plane $\operatorname{Re}\beta>0$. As recently proved by Wagner [7], if the pair potential is bounded and absolutely integrable, the correlation functions and the pressure turn out to have Borel summable Taylor expansions at $\beta=0$ (for Borel summability, see e.g. [4, 6]). Among other facts the proof uses analyticity for $\operatorname{Re}\beta>0$ and the bound $\int|\Phi(x)|^n dx\leqq(\|\Phi\|_\infty)^{n-1}\|\Phi\|_1$.

Here the aim is to prove the Borel-Le Roy summability ([3, 2]) of these power series, under suitable hypotheses on the pair potential $\Phi(r)$. Hypotheses (1), (2), (3) below include, in particular, the asymptotic behaviour $\Phi(r)\sim(\ln r^{-1})^p$ as $r\to 0$ $(p\in\mathbb{N})$. These assumptions allow us to analytically continue the correlation functions beyond the right half plane, to a region containing $\left\{\beta/\operatorname{Re}\beta^{\frac{1}{1+p}}>0\right\}$ on the Riemann surface of $\ln\beta$, which is suggested by the analytic structure of $\int(e^{-\beta\Phi(x)}-1)dx$ in these cases (Proposition 2.1). Moreover the power series remainders are proved not to grow faster than $((p+1)n)!$, which is somehow suggested by bounds of the type $\int|\Phi(x)|^n dx\leqq c(pn)!$, and by a further factor $(n!)^2$ that can be expected in the estimates of n^{th} derivatives of correlation functions.

In the case $\nu=2$, $p=1$, conditions (1), (2), (3) include potentials exponentially decreasing as $r\to+\infty$ and with the asymptotic behaviour of two-dimensional Yukawa potentials (see e.g. [8, 1]) as $r\to 0$, although $\Phi(r)=e^{-ar}(\ln r^{-1})$ is not in this