Commun. Math. Phys. 98, 105–117 (1985)

Communications in Mathematical Physics

De Sitter Superalgebras and Supergravity*

K. Pilch[1]**, P. van Nieuwenhuizen[1], and M. F. Sohnius[2]***

1 Institute for Theoretical Physics, State University of New York at Stony Brook, Stony Brook, Long Island, NY 11794, USA

2 Department of Applied Mathematics and Theoretical Physics, University of Cambridge, Cambridge CB3 9EW, England

Abstract. A general analysis of all possible super-extensions of anti-de Sitter and de Sitter algebras $O(3,2)$ and $O(4,1)$ is presented. It is shown that actions with de Sitter local supersymmetry exist, but contain vector-ghosts.

1. Introduction

Classical solutions of supergravity models with cosmological constants have been constructed from anti-de Sitter metrics with space-time symmetry $O(3,2)$, but not from de Sitter metrics with $O(4,1)$. A number of arguments are usually put forward for the non-existence of supergravity models with a positive cosmological constant. Such arguments are often based on the *non-existence of Majorana spinors* for $O(4,1)$. Indeed, one can use the "Noether coupling" approach to supergravity to directly show that Majorana gravitini are incompatible with a positive cosmological constant: the cosmological term in the Lagrangian, $a_1\sqrt{g}$, is accompanied by a gravitino mass term $a_2\sqrt{g}\,\bar{\psi}_\mu\gamma^{\mu\nu}\psi_\nu$ and by a term $\delta\psi_\mu = D_\mu\varepsilon + a_3\gamma_\mu\varepsilon + \ldots$ in the gravitino transformation law. Demanding invariance of the Lagrangian, one finds relationships $a_1 = a_2 a_3$ and $a_2 = a_3$. The Majorana property of ψ_μ fixes a_3 to be real, with the result that $a_1 \geqq 0$. The details of this are dependent on notation and conventions, but the result is not.

There clearly is a way out of this type of no-go situation, as there is no need to insist on the existence of Majorana spinors. We may simply accept that for every spinor its charge-conjugate is also present and independent. The usual rules for counting spinors in supergravity then mean that in the de Sitter case we must have extended supergravity with *even* N. Once we have included all charge-conjugates in the basic set of N spinors, there will be a symplecitic Majorana condition

$$(Q_{\alpha i})^* = E_i{}^j D_\alpha{}^\beta Q_{\beta j} \tag{1}$$

for the de Sitter case, with $E^T = -E$, rather than a straightforward one with $E = \mathbf{1}$.

* Supported in part by the NSF under grant PHY 81-09110 A-01

** On leave from the Institute of Theoretical Physics, University of Wrocław, Wrocław, Poland

*** Address from Oct. 1984: Imperial College, London SW7 2BZ, England