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Symmetries of Einstein-Yang-Mills Fields and Dimensional Reduction

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Abstract. Let E be a manifold on which a compact Lie group S acts simply (all orbits of the same type); E can be written locally as $M \times S/I$, M being the manifold of orbits (space-time) and I a typical isotropy group for the S action. We study the geometrical structure given by an S-invariant metric and an S-invariant Yang Mills field on E with gauge group R. We show that there is a one to one correspondence between such structures and quadruplets $(\gamma_{\mu\nu}, A^{\hat{\alpha}}_{\mu}, \phi^i_{\alpha}, h_{\alpha\beta})$ of fields defined solely on M; $\gamma_{\mu\nu}$ is a metric on M, $h_{\alpha\beta}$ are scalar fields characterizing the geometry of the orbits (internal spaces), ϕ^i_{α} are other scalar fields (Higgs fields) characterizing the S invariance of the Lie(R)-valued Yang Mills field and $A^{\hat{\alpha}}_{\mu}$ is a Yang Mills field for the gauge group $N(I)|I \times Z(\lambda(I))$, $N(I)$ being the normalizer of I in S, λ is a homomorphism of I into R associated to the S action, and $Z(\lambda(I))$ is the centralizer of $\lambda(I)$ in R. We express the Einstein-Yang-Mills Lagrangian of E in terms of the component fields on M. Examples and model building recipes are given.

I. Introduction

I.1. Several Descriptions for the Same Geometrical Structure

Symmetry properties of gravity (metric structure) and Yang-Mills fields (connections) have been often studied separately, both by physicists and mathematicians. These two kinds of geometrical structures are however deeply inter-related and several techniques of "dimensional reduction" allow us to cast a new light on the subject. Let us suppose that we live in an extended universe U endowed with a metric $g(U)$ invariant under a group G (description 1), then, in many cases, we can also describe the same situation by saying that we live in an universe E ($\dim E < \dim U$) endowed with a metric $g(E)$ and a Yang-Mills field $A(E)$, both invariant under a subgroup of G (description 2). We can finally describe the same

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