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Convergence of Migdal-Kadanoff Iterations in Non-Abelian Lattice Gauge Models

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Abstract. We study the Migdal-Kadanoff recursion relations for lattice gauge models with gauge groups SU(N) or U(N) in dimensions $d<4$. It is shown that the Gibbs factor of a plaquette with Wilson action is driven to 1 for all values of the "temperature" (coupling constant). For models recently proposed by K.R. Ito, where Migdal's and Kadanoff's recursion relations hold exactly, a lower bound on the string tension is derived. The results obtained by us extend those derived by Ito for U(1). Our method is based on analytic continuation of the Gibbs factors, which are class functions, in the central angles.

1. Introduction

The recursion relations of Migdal [1] and the modified ones of Kadanoff [2] have been proposed as approximate real space renormalization group transformations both for spin systems and lattice gauge theories. However, only very recently these recursion relations have been investigated by purely analytic methods [3, 4]. Ito [4] studied the U(1) gauge group with Villain and Wilson action. For dimensions $d<4$ he showed that the effective actions generated by both types of recursion relations are always driven to the high temperature (strong coupling) region. Moreover he proposed special lattice gauge theory models in which the recursion relations of Migdal and of Kadanoff hold exactly and derived for $d=3$ a lower bound for the U(1) string tension of these models.

In this article we generalize Ito's work to the non-abelian gauge groups SU(N) and U(N). Migdal's [M] and Kadanoff's [K] recursion relations involve multiple convolutions of class functions on the gauge group G, $n\in\mathbb{N}_0$,

$$g^{(n+1)}(u)=\left\{\frac{g^{(n)*r}(u)}{g^{(n)*r}(e)}\right\}^q, \tag{1.1M}$$

$$g^{(n+1)}(u)=\frac{((g^{(n)})^q)^{*r}(u)}{((g^{(n)})^q)^{*r}(e)}, \tag{1.1K}$$

with q,r positive integers, $u\in G$, and e the unit element of G.