

Relation Between Effective Couplings for Asymptotically Free Models

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Abstract. For asymptotically free models with two independent couplings asymptotic expansions are constructed which express one effective coupling in terms of the other. The expansions involve powers (including fractional or irrational exponents) and logarithms. All orders of the β -functions are taken into account. The expansions found are complete in the sense that they represent solutions (exact to any order) which generalize all the solutions obtained with the β -functions approximated to second order. It is shown that higher orders are relevant since it is not possible in general to reparametrize the system such that the β -functions become polynomials of the coupling parameters. The simplifications in case of supersymmetric models are discussed.

1. Introduction

In this paper asymptotic properties of effective couplings will be studied for massless field theoretical models which are asymptotically free and involve two coupling constants. As example may serve a non-Abelian gauge field of coupling constant g to which a Higgs field with interaction constant λ is coupled. The effective coupling parameters \bar{g} and $\bar{\lambda}$ are defined as functions of the coupling constants, a Euclidean momentum variable $k^2 < 0$ and a normalization mass $\kappa^2 < 0$. In terms of dimensionless variables,

$$\bar{g} = \bar{g}(u, g, \lambda), \quad \bar{\lambda} = \bar{\lambda}(u, g, \lambda), \quad u = \frac{k^2}{\kappa^2}. \quad (1.1)$$

A model is called asymptotically free if both effective couplings vanish in the limit of large Euclidean momenta [1–3]

$$\lim_{u \rightarrow \infty} \bar{g} = 0, \quad \lim_{u \rightarrow \infty} \bar{\lambda} = 0. \quad (1.2)$$

Only solutions with bounded ratio $\bar{\lambda}/\bar{g}^2$ will be considered.