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Construction of Canonical Coordinates on Polarized Coadjoint Orbits of Lie Groups

S. A. Kamalin and A. M. Perelomov

Institute of Theoretical and Experimental Physics, SU-117259 Moscow, USSR

Abstract. Construction of canonical coordinates on polarized coadjoint orbits of Lie groups is presented.

0. Introduction

Hamiltonian systems on orbits of coadjoint representation of Lie groups have been studied in numerous papers [1–8]. The general theory of such systems developed in refs. [1–4, 8] enables one to indicate cases of complete integrability. However, to explicitly describe a Hamiltonian system one should be able to introduce canonical coordinates on a requisite orbit. In the simplest case of the fourth-order matrices such coordinates were introduced in the paper by Symes [5] who used the M. Vergne algorithm [9]. This algorithm is applicable for any completely solvable Lie group but practically it turns out to be very cumbersome.

This paper is an extended version of the preceding note [7]. We will give in it a simple construction of canonical coordinates for orbits possessing polarization. Some polarizations are shown for graded Lie groups. Making use of this method one can explicitly parametrize the orbits of coadjoint representation of the Borel subgroups of the real split Lie groups and describe the corresponding Hamiltonians.

I. Polarizable Ad*-Orbits

In this section we construct canonical coordinates on polarizable orbits of coadjoint representation of Lie groups. Let us recall some definitions.

Let $\mathfrak{G}^*$ be a space dual to Lie algebra $\mathfrak{G}$, then for functions $f, g \in \mathscr{F} = C^\infty(\mathfrak{G}^*)$ the Poisson bracket

$$\{f, g\}(x) = \langle x, [\nabla f(x), \nabla g(x)]\rangle$$

is defined which continues the commutator in Lie algebra $\mathfrak{G}$ considered as a space of linear functions on $\mathfrak{G}^*$ ($\langle x, \xi\rangle$ stands for the value of functional $x \in \mathfrak{G}^*$ on element $\xi \in \mathfrak{G}$). The kernel of form $\langle x, [.,.]\rangle$ coincides with the isotropy