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The Cauchy Problem in Extended Supergravity, $N=1$, $d=11$

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Abstract. We prove the Grassmann valued system of extended supergravity $N=1$, $d=11$ proposed by Cremmer and Julia is well proposed and causal.

Introduction

The extension of supergravity to d-dimensional models offers a synthesis between ordinary $d=4$, $N=1$ supergravity [8, 14] which unifies a bosonic and a fermionic field (graviton and gravitino) and the old ideas of Kaluza-Klein-Jordan-Thiry for unification of gravitation and electromagnetism through a fifth dimension of space time, an idea extended to Yang-Mills fields by B. DeWitt, R. Kerner etc. Among the various extended supergravity models proposed for unification of all fundamental interactions a particularly interesting one, called $N=1$, $d=11$ supergravity, is an Einstein Cartan theory in an 11 dimensional space time with source a spin 3/2 field, a spinor valued 1-form. However, it is necessary, in order to have a coherent system, to add another field called the "three index photon", a numerical valued 3-form.

We show in this paper – as we have done before for simple supergravity cf. [2], that the system of partial differential equations of the $N=1$, $d=11$ extended supergravity satisfied by the Grassmann valued fields is a well posed system for the Cauchy problem, with constraints but causal: the solution at a point depends only on the initial data which are in the past of that point, this past being determined by the isotropic cone of the numerical part of the metric.

1. Notations

$V=S\times\mathbb{R}$, 11 dimensional, C^∞ manifold, x^M, $M=0,\ldots,10$ local coordinates, $\partial_M=\partial/\partial x^M$, $\mathbf{e}=(e_A{}^M)$, $\mathbf{e}_A=e_A{}^M\partial_M$: 11 dimensional moving frame, $e^A{}_M$ inverse matrix of $e_A{}^M$, $\theta^A=e^A{}_M dx^M$ moving coframe dual of $\mathbf{e}_A$.

$$\begin{aligned} &g_{MN}=e^A{}_M e^B{}_N \eta_{AB}: \quad \text{hyperbolic metric } \mathbf{g}, \\ &\eta_{AB}=\mathrm{diag}(1,-1,\ldots,-1) \quad \text{Minkowski metric}. \end{aligned} \tag{1.1}$$