

# Min-Max Theory for the Yang-Mills-Higgs Equations

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**Abstract.** In each monopole sector there exist an infinite number of finite energy solutions to the Prasad-Sommerfield limit of the SU(2) Yang-Mills-Higgs equations on  $\mathbb{R}^3$  whose energy is greater than any finite number.

## Contents

|   |     |
|---|-----|
| A.1 Introduction . . . . .                                      | 473 |
| A.2 Strategy for Convergence . . . . .                          | 476 |
| A.3 Terminology . . . . .                                       | 482 |
| A.4 The Variational Problem . . . . .                           | 483 |
| B.1 The Regularized Problem . . . . .                           | 486 |
| B.2 Minimizing Sets for $\mathfrak{A}^\delta$ . . . . .         | 488 |
| B.3 Convergence for Min-Max for $\mathfrak{A}^\delta$ . . . . . | 493 |
| B.4 Final Arguments for $\mathfrak{A}^\delta$ . . . . .         | 496 |
| C.1 A priori Estimates . . . . .                                | 498 |
| C.2 Exponential Decay Estimates . . . . .                       | 502 |
| C.3 Power Law Estimates for $\Phi$ . . . . .                    | 505 |
| C.4 Power Law Estimates for $A$ . . . . .                       | 508 |
| D.1 The Stress Energy Identity . . . . .                        | 513 |
| D.2 The Cluster Decomposition . . . . .                         | 516 |
| D.3 The Interaction of Clusters . . . . .                       | 519 |
| D.4 The Convergence of Min-Max for $\mathfrak{A}$ . . . . .     | 526 |
| E.1 The Neighborhood of the Moduli Space . . . . .              | 528 |
| E.2 Min-Max, and Pointed Homotopy . . . . .                     | 537 |

## A.1. Introduction

The differential equations of a classical gauge theory are, in many cases, the formal variational equations of a functional (the action) on a topologically non-trivial space. And so it was conjectured [1, 2] that Morse theory, or some weaker analog might be useful for establishing the existence of non-trivial solutions. There are

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