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Chiral Anomalies in Even and Odd Dimensions

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Abstract. Odd dimensional Yang–Mills theories with an extra ‘topological mass” term, defined by the Chern–Simons secondary characteristic, are discussed. It is shown in detail how the topological mass affects the equal time charge commutation relations and how the modified commutation relations are related to non-abelian chiral anomalies in even dimensions. We also study the SU(3) chiral model (Wess-Zumino model) in four dimensions and we show how a gauge invariant interaction with an external SU(3) vector potential can be defined with the help of the Chern–Simons characteristic in five dimensions.

1. Introduction

In a Yang–Mills theory in $2+1$ space–time dimensions one can add a “topological mass” term $\alpha\mathscr{L}_{\mathrm{cs}}^{(3)}$ to the Yang–Mills Lagrangian $\mathscr{L}_{\mathrm{ym}}$ such that the field equations remain gauge invariant and describe a gauge field with mass α, [1]. The extra term $\mathscr{L}_{\mathrm{cs}}^{(3)}$ is known also as the Chern–Simons secondary characteristic and a general formula, valid in all odd dimensions, can be found in [2]. The Lagrangian $\mathscr{L}=\mathscr{L}_{\mathrm{ym}}+\alpha\mathscr{L}_{\mathrm{cs}}^{(3)}$ is not gauge invariant and it defines a phase factor $\Lambda(U,A)$ by $S(U^{-1}AU+U^{-1}\,dU)=\Lambda(U,A)\cdot S(A)$, where $S(A)=\exp\int i\mathscr{L}(A)\,d^3x$ is the action as a function of the vector potential A. As shown in [3], the phase factor Λ modifies the local charge commutation relations (provided one has non-trivial boundary conditions for the gauge fields at space infinity) in such a way that the new commutators define an one dimensional central extension of the Lie algebra of infinitesimal gauge transformations (one has “Schwinger terms”), isomorphic to a Kac-Moody algebra. The central extension is determined *by the boundary values* at $\|\vec{x}\|\to\infty$ of the gauge fields; if we require zero boundary conditions then the extension is trivial.

In the present paper I shall extend the results to $2n+1$ dimensions and the relation between the modified current algebra in $2n+1$ dimensions and non-abelian chiral anomalies in $2n$ dimensional field theories is investigated. The origin of chiral anomalies is not discussed in this paper; for a physical background a reader not familiar with anomalies is suggested to look at [5] or [6].