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Reduction in the Number of Coupling Parameters

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Dedicated to the memory of Kurt Symanzik

Abstract. A method is developed for reducing the formulation of massless models with several independent couplings to a description in terms of a single coupling parameter. The original as well as the reduced system are supposed to be renormalizable and invariant under the renormalization group. For most models there are, if any, only a finite number of reductions possible including those which correspond to symmetries of the system. The reduction method leads to a consistent formulation of the reduced model in any order of perturbation theory even in cases where it is difficult to establish a symmetry in higher orders. An example where no symmetry seems to be involved is the interaction of a spinor field with a pseudoscalar field. For this model the reduction method determines the quartic coupling constant uniquely as a function of the Yukawa coupling constant. The Wess-Zumino model is an exceptional case for which the reduction method admits an infinite number of solutions besides the supersymmetric one.

1. Introduction

Symmetry considerations provide a natural method of reducing the number of independent parameters in models of quantum field theory. If a symmetry is imposed, otherwise unconstrained coupling parameters become related among each other so that the number of independent parameters is decreased. Renormalizability of the model is maintained provided anomalies are absent and the symmetry can be implemented in all orders of perturbation theory.

In this paper a more general approach for reducing the number of coupling parameters is taken which is based on the principles of renormalizability and invariance under the renormalization group. It turns out that these requirements severely limit the possibilities of constraining the coupling parameters to a single independent one. The method is developed for the reduction of massless models from $n+1$ coupling parameters $\lambda_0, \lambda_1, \ldots, \lambda_n$ to a description in terms of λ_0 only. Any symmetry requirement leading to a renormalizable formulation is certainly