Commun. Math. Phys. 96, 539–565 (1984)

Communications in
Mathematical
Physics

Non-Abelian Magnetic Monopoles*

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Abstract. It is shown that a general, irreducible, SU(n), Sp(n), SO($2n$) monopole with maximal symmetry breaking is determined by its spectral data. For SU(n) with minimal symmetry breaking the spectral data is defined and also shown to determine the monopole.

Introduction

In a previous paper [12] the definition of the spectral curve of a monopole, given in [9] by Hitchin for SU(2), was extended to any compact, connected, simple Lie group K. In this paper the details of the results announced in [12] are presented. It is shown that there are $r = \operatorname{rank} K$ spectral curves $S_1, \ldots, S_r$ for a K monopole. The spectral curves are labelled by the simple roots $\{\alpha_1, \ldots, \alpha_r\}$, and when α_i and α_j are joined on the Dynkin diagram the intersection $S_i \cap S_j$ has a splitting as $S_i \cap S_j = S_{ij} \cup S_{ji}$. The curves and this splitting constitute the *spectral data* of the monopole. The main result of this paper is that for SU(n), SO($2n$) and Sp(n) an irreducible, general monopole is determined by its spectral data.

In Sect. 1 the basic material on monopoles and the definition of the magnetic charges $\{m_1, \ldots, m_r\}$ of a monopole, with maximal symmetry breaking at infinity, are reviewed. The definition of the twistor space $\mathscr{T}$ is also recalled and used in Sect. 2 to generalize the twistor correspondence of Hitchin and Ward. The general twistor correspondence associates to any K monopole with reduction at infinity to a maximal torus T, a holomorphical principal bundle Q, on $\mathscr{T}$, with structure group G, the complexification of K, and two reductions $R^+, R^- \subset Q$ to Borel subgroups of G.

* Research supported in part by NSF Grant 8120790

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