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Fermion Ito's Formula and Stochastic Evolutions*

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Abstract. An Ito product formula is proved for stochastic integrals against Fermion Brownian motion, and used to construct unitary processes satisfying stochastic differential equations. As in the corresponding Boson theory [10, 11] these give rise to stochastic dilations of completely positive semigroups.

1. Introduction

In [10] a quantum stochastic calculus leading to an Ito product formula was developed which, in its simplest form, uses as integrators the Boson field operators

$$A(t)=a(\chi_{[0,t]}), \qquad A^\dagger(t)=a^\dagger(\chi_{[0,t]}). \tag{1.1}$$

Here χ_S denotes the indicator function of the set S and the operators (1.1) are the smeared fields corresponding to $\chi_{[0,t]}$ living in the Boson Fock space over the Hilbert space

$$\mathcal{h}=L^2[0,\infty). \tag{1.2}$$

Under the duality transformation this Fock space transforms into the L^2-space of Wiener measure in such a way that $A(t)+A^\dagger(t)$ becomes multiplication by Brownian motion; thus the operators (1.1) constitute a quantum Brownian motion [6]. The Ito product formula can be summarized by the multiplication rules for stochastic differentials

	dA	$dA^\dagger$	dt
dA	0	dt	0
$dA^\dagger$	0	0	0
dt	0	0	0

(1.3)

which contain the classical Ito formula as a special case [10].

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