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Random Surface Correlation Functions

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Abstract. Truncated pair functions for free random surface models and Bernoulli ensembles are examined. In both cases, the pair function is shown to obey Ornstein-Zernike scaling whenever various correlation lengths of the system satisfy a nonperturbative criterion. Under the same conditions, the transverse displacement of surfaces contributing to the pair function is shown to be normally distributed. A new type of transition, which concerns the width of typical surfaces, is introduced and studied. Whenever the system is below the melting transition temperature of a related lower-dimensional model, the width of typical surfaces is shown to be finite. A thermodynamic formalism for free random surface models is developed. The formalism is used to obtain sharp estimates of the entropy of surfaces contributing to the pair function.

1. Introduction

The stochastic geometry of random surfaces has recently become a topic of considerable interest (see [1] and references therein). The correlation functions of lattice gauge theories, three-dimensional spin systems and models of crystalline interfaces have natural expressions as weighted sums over surfaces. However, such expressions are difficult to analyze due to both the combinatoric problems introduced by the large number of surfaces, and the intractability of explicit forms for the associated weights. It is therefore of interest to study models of correlation functions which are *defined* as sums over restricted classes of surfaces with relatively simple weights.

In this paper, we analyze the beahvior of correlation functions of the form

$$Q_{\mathscr{S}}(\beta) = \sum_{S \in \mathscr{S}} e^{-\beta|S|}, \tag{1.1}$$

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