Commun. Math. Phys. 96, 431–437 (1984)

Communications in
Mathematical
Physics

Solutions to Yang-Mills Field Equations in Eight Dimensions and the Last Hopf Map

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Abstract. We will show that the Hopf map $S^{15} \xrightarrow{S^7} S^8$ admits a sourceless, topologically non-trivial gauge field. This result will be cast in the form of a solution to eight dimensional Euclidean Yang-Mills field equations with topological charge $Q=1$. This solution is Spin(9) symmetric and leads to a new generalized duality condition $F \wedge F = \pm (F \wedge F)^*$.

It is becoming increasingly apparent that many of the subtleties of field theories can be understood only by investigating their global properties. This has necessarily led to the introduction of several new mathematical results and techniques into the physics literature, which has in turn prompted mathematicians in the areas of algebraic and differential topology to investigate a number of system of importance to physics. The most striking examples of this interdisciplinary pursuit are instanton solutions to the $D=4$ dimensional Yang-Mills field equations [1].

In this paper we will study the higher dimensional analog of the instanton, but first a few words of background are in order.

Trautman [2] was the first to realize that the one instanton solution to the SU(2) $D=4$ Euclidean Yang-Mills field equations is related to a Hopf map [3]

$$S^7 \xrightarrow{S^3} S^4$$

i.e., the mapping associated with a principal fibre bundle structure with base space S^4 (the one point compactification of Euclidean four-space) and fibre $SU(2) \sim S^3$. The canonical connection on this bundle defines a self-dual or anti-self-dual curvature tensor $F_{\mu\nu}$.

Nowakowski and Trautman [2] consider more generally the natural connection on Stiefel bundles, and Laquer [13] considers the Yang-Mills functional associated with principal bundles over homogeneous spaces.