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Splitting Theorems for Spatially Closed Space-Times

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Abstract. A Lorentzian splitting theorem is obtained for spatially closed space-times. The proof employs and extends some recent results of Bartnik and Gerhardt concerning the existence and rigid uniqueness of compact maximal hypersurfaces in spatially closed space-times. A splitting theorem for spatially closed *time-periodic* space-times, which generalizes a result first considered by Avez, is derived as a corollary.

1. Introduction

Yau [12] has posed the problem of establishing a Lorentzian splitting theorem analogous to the splitting theorem of Cheeger and Gromoll [5] for Riemannian manifolds. In this paper we prove the following splitting result for spatially closed space-times.

Theorem 1.1. *Let V be a space-time which has the following properties:*

(A) *V contains a compact Cauchy surface.*

(B) *V satisfies the timelike convergence condition, i.e.,* $\mathrm{Ric}(X,X)\geqq 0$ *for all timelike X.*

(C) *V contains a timelike curve which is future and past complete.*

(D) *For each $p\in V$, every future (past) inextendible null geodesic η issuing from p reaches a point in the timelike future (past) of p, i.e., $\eta\cap I^+(p)\neq\phi(\eta\cap I^-(p)\neq\phi)$.*

Then V splits into the pseudo-Riemannian product of $(\mathbb{R}, -dt^2)$ *and* (M,h), *where M is a smooth compact spacelike hypersurface and h is the induced metric on M. In particular if V is Ricci flat and* $\dim V=4$ *then V is flat.*

Remarks. We shall always use the term “hypersurface” to mean “hypersurface without boundary.” Put more succinctly, condition (D) states that there exists a null cut point along each future and past inextendible null geodesic. In Sect. 3 it is shown that for space-times admitting a compact Cauchy surface, (D) is equivalent to the requirement that there be no observer with a nontrivial future or past event horizon.