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Conformal Symmetry in Two Dimensions: An Explicit Recurrence Formula for the Conformal Partial Wave Amplitude

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Abstract. An explicit recurrence relation for the conformal block functions is presented. This relation permits one to evaluate the X-expansion of these functions order-by-order and appropriate for numerical calculations.

The properties of infinite algebra of infinitesimal conformal transformations of two-dimensional space-time (its central extension is known as Virasoro algebra) and its consequences for field theory are now under extensive investigation [1, 3]. In this theory an important role belongs to the so-called conformal block functions, or "conformal partial amplitudes" [1], which are essentially sums over the "S-channel" contributions of all conformal fields of the same conformal class to the four-point function of a certain set of conformal fields (see ref. 1). For example, the associativity property of operator algebra in conformal field theory can be expressed in terms of these functions as a set of conformal bootstrap equations [1, 2]. So, the solution of the conformal bootstrap equations (e.g. numerical) requires an effective method to calculate conformal block functions.

In principle these functions could be evaluated straightforwardly as a series in powers of anharmonic ratio X of the correlation function, solving level-by-level in an appropriately chosen basis the following set of equations in the conformal module space, i.e., the space spanned by all operators of the same conformal class of dimension D:

$$L(k)|n+k) = (D + kd_1 - d_2 + n)|n)\,, \tag{1}$$

where $L(k)$ are Virasoro generators of infinitesimal conformal transformations and $|n)$ is the n^{th} level contribution to the state

$$V_{d_1}(x)V_{d_2}(0)|0) = x^{D-d_1-d_2}\sum x^n|n)\,, \tag{2}$$

which is the intermediate state in the characteristic function:

$$F(D, d_i, C, x) = (0|V_{d_3}(\infty)V_{d_4}(1)V_{d_1}(x)V_{d_2}(0)|0)\,. \tag{3}$$