

Gravitational Anomalies and the Family's Index Theorem*

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Abstract. We discuss the use of the family's index theorem in the study of gravitational anomalies. The geometrical framework required to apply the family's index theorem is presented and the relation to gravitational anomalies is discussed. We show how physics necessitates the introduction of the notion of *local cohomology* which is distinct from the ordinary topological cohomology. The recent results of Alvarez-Gaumé and Witten are derived by using the family's index theorem.

I. Introduction

Alvarez-Gaumé and Witten [1] have calculated the gravitational anomalies of certain parity violating theories in $4k - 2$ dimensions. Their most striking result is that there is a unique minimal ten dimensional theory where the gravitational anomalies cancel. In this communication we reproduce their results in a different way by using the family's index theorem [2] instead of Feynman diagram methods.

The relation of the family's index theorem to anomalies has been discussed by Atiyah and one of the present authors in reference [3]. In that paper, the geometric setting for the family's index theorem was presented and the relation to anomalies was discussed. The authors showed that the first characteristic class of the index bundle for the Dirac operator was related to anomalies. A number of papers have addressed the relationships among chiral anomalies, the geometry of the space of vector potentials, and the families of Dirac operators. We recommend the papers of Alvarez-Gaumé and Ginsparg [4], Lott [5], and Stora [6] to the reader. The first investigation of the behavior of the Dirac operator as a function of the metric is due to Hitchin [7].

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