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The Complete Set of Hamiltonian Intermittency Scaling Behaviors

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Abstract. Renormalization group equations describing the phenomenon of intermittency in Hamiltonian systems are presented. All solutions satisfying certain physical constraints are obtained; they are the complete set of simple singularities. Further considerations lead to precise predictions for scaling behavior at the onset of intermittency.

I. Introduction

It has long been known that a Hamiltonian system with one degree of freedom, $H(p,q)$, has particularly simple behavior near an elliptic or hyperbolic fixed point. A canonical transformation achieves Birkhoff's normal form

$$H(p,q)=a\left(\frac{p^2+q^2}{2}\right)+b\left(\frac{p^2+q^2}{2}\right)^2+c\left(\frac{p^2+q^2}{2}\right)^2+\cdots \tag{1}$$

in the neighborhood of an elliptic fixed point, or the normal form

$$H(p,q)=apq+b(pq)^2+c(pq)^3+\cdots \tag{2}$$

in the neighborhood of a hyperbolic fixed point [1, 2]. What is the corresponding normal form in the limit of marginal stability ($a=0$)? An answer to this question is found in the singularity theory of Arnold [3]. By limiting consideration to a special type of behavior, called "simple," he obtains a discrete classification of the possibilities. The results are not widely known or understood by physicists, perhaps because of the unfamiliar mathematical techniques involved. Presented in this paper is a physically motivated calculation of Arnold's simple normal forms, based on the renormalization group for mappings introduced by Feigenbaum [4, 5].

Feigenbaum's renormalization group for mappings has been used to study three phenomena occurring in Hamiltonian systems: infinite cascades of period-doubling bifurcations, the breakup of KAM tori and tangent bifurcations [6–11, 23–25]. The classification of simple normal forms is obtained through a comprehensive treatment of the latter. The physical motivation for studying tangent bifurcations is