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Quantum Logic, State Space Geometry and Operator Algebras

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Abstract. The problem of characterising those quantum logics which can be identified with the lattice of projections in a JBW-algebra or a von Neumann algebra is considered. For quantum logics which satisfy the countable chain condition and which have no Type I_2 part, a characterisation in terms of geometric properties of the quantum state space is given.

Introduction

Quantum logics, as defined below, are σ-complete orthomodular lattices. They have been vigorously investigated in recent years. In most mathematical formulations of the foundations of quantum mechanics the lattice of "questions" associated with a physical system is a quantum logic.

Important examples of quantum logics are, in order of successive generalisation:

(a) The lattice of all closed subspaces of a separable Hilbert space.
(b) The lattice of all projections in a von Neumann algebra.
(c) The lattice of all projections in certain Jordan operator algebras known as JBW-algebras.

Characterisation of those quantum logics isomorphic to (a) have been obtained by Piron, in 1964, (see [8]), and by Wilbur [9], in 1977. Can one characterise those quantum logics isomorphic to the lattice of all projections in a von Neumann algebra, or in a JBW-algebra, by geometric properties of the quantum state space of a quantum logic?

We obtain a partial solution to this problem by restricting our attention to quantum logics which satisfy the countable chain condition and which have no Type I_2 part (see below for definitions). We show that, when Q is such a quantum logic, there are three geometric properties which will be satisfied by the quantum state space of Q if, and only if, Q is isomorphic to the lattice of all projections in a JBW-algebra.

We also, as a corollary, give a geometric characterisation of those orthomodular