Commun. Math. Phys. 96, 223–250 (1984)

Communications in Mathematical Physics

Propagators and Renormalization Transformations for Lattice Gauge Theories. II

T. Bałaban*

Department of Physics, Harvard University, Cambridge, MA 02138, USA

Abstract. We continue the studies of the Paper I and extend the results of this paper to operators defined by restrictions on different scales, or by renormalization transformations of different orders.

Introduction

In our study of ultra-violet stability for gauge field theories we will have to consider propagators defined by some quadratic forms with restrictions of different scales on disjoint subdomains of a lattice. In this paper we will continue the investigation of the fundamental quadratic form $\langle \partial A, \partial A \rangle$, and the restrictions are given by averaging operators of different orders, introduced in the first part [4]. We will be interested in the same properties as before, thus in local regularity properties and in exponential decay, but we will have to relate them to many scales appearing in the problem.

This paper is a continuation of [4] and we use all the notations introduced there without further explanation. We refer to the results and the formulas of this paper using its original numbering, for example Proposition 1.3, formula (1.81), and so on. The only notation we would like to mention here is that a distance between two points $x, y \in \eta Z^d$, $\eta > 0$, is given by the l^1-norm of the vector $x-y$ and is denoted by $|x-y|$, i.e. $|x-y| = \sum_{\mu=1}^{d} |x_\mu - y_\mu|$. This distance depends of course on the scale of the lattice.

A. Operators Defined by Conditions on Many Scales

Let us begin with a generalization of the variational problem considered in Sect. D of [4] and leading to the operator H_k. At first we have to describe a geometry of

* Research supported in part by the National Science Foundation under Grant PHY-82-03669