

Computation Theory of Cellular Automata

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Abstract. Self-organizing behaviour in cellular automata is discussed as a computational process. Formal language theory is used to extend dynamical systems theory descriptions of cellular automata. The sets of configurations generated after a finite number of time steps of cellular automaton evolution are shown to form regular languages. Many examples are given. The sizes of the minimal grammars for these languages provide measures of the complexities of the sets. This complexity is usually found to be non-decreasing with time. The limit sets generated by some classes of cellular automata correspond to regular languages. For other classes of cellular automata they appear to correspond to more complicated languages. Many properties of these sets are then formally non-computable. It is suggested that such undecidability is common in these and other dynamical systems.

1. Introduction

Systems that follow the second law of thermodynamics evolve with time to maximal entropy and complete disorder, destroying any order initially present. Cellular automata are examples of mathematical systems which may instead exhibit “self-organizing” behaviour¹. Even starting from complete disorder, their irreversible evolution can spontaneously generate ordered structure. One coarse indication of such self-organization is a decrease of entropy with time. This paper discusses an approach to a more complete mathematical characterization of self-organizing processes in cellular automata, and possible quantitative measures of the “complexity” generated by them. The evolution of cellular automata is viewed as a computation which processes information specified as the initial state. The structure of the output from such information processing is then described using

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¹ An introduction to cellular automata in this context, together with many references is given in [1]. Further results are given in [2, 3], and are surveyed in [4, 5]