Commun. Math. Phys. 95, 487–532 (1984)

Communications in Mathematical Physics

Renormalization of Dyson's Hierarchical Vector Valued ϕ^4 Model at Low Temperatures

P. M. Bleher[1] and P. Major[2]

1 Institute of Applied Mathematics of the Soviet Academy of Sciences, Moscow, USSR
2 Mathematical Institute of the Hungarian Academy of Sciences, Budapest, Hungary

Abstract. We investigate Dyson's hierarchical vector valued ϕ^4 model at low temperatures. The case $2 > c > \sqrt{2}$ is considered. The pure phase is constructed, and the existence of its large scale limit is proved. The limit is Gaussian, but an unusual normalization has to be chosen. In the direction of the spontaneous magnetization one has to divide by the square root of the volume, but in the orthogonal direction one has to divide by a different power of the volume for all low temperatures.

1. Introduction

In this paper Dyson's hierarchical vector valued ϕ^4 model is investigated at low temperatures. First we describe the model we are working with (see [2, 7]).

We define the volumes $V_{k,n}$, $V_{k,n} \subset \mathbb{Z}$, $\mathbb{Z} = \{1, 2, \ldots\}$ as $V_{k,n} = \{j, j \in \mathbb{Z}, (k-1)2^n < j \leqq k \cdot 2^n\}$, $n = 1, 2, \ldots, k = 1, 2, \ldots$. Put $V_{1,n} = V_n$. For $i, j \in \mathbb{Z}$ we set

$$n(i,j) = \min\{n, \textit{there exists a } k \text{ such that } i \in V_{k,n}, j \in V_{k,n}\}.$$

The hierarchical distance $d(i,j)$, $i, j \in \mathbb{Z}$, is defined as

$$d(i,j) = \begin{cases} 0 & \text{if } i = j \\ 2^{n(i,j)-1} & \text{if } i \neq j. \end{cases}$$

The spins $\sigma(i)$, $i \in \mathbb{Z}$, take on values in the m-dimensional Euclidean space R^m. The energy of a configuration $\sigma = \{\sigma(i), i \in V_{k,n}\}$ is defined by the formula

$$H_{k,n}(\sigma) = \sum_{\substack{(i,j), i \neq j \\ i,j \in V_{k,n}}} U(i,j)(\sigma(i); \sigma(j)), \tag{1.1}$$

where $U(i,j) = -d^{-\alpha}(i,j)$ and $(\cdot;\cdot)$ denotes scalar product. In particular

$$H_n(\sigma) = \sum_{\substack{(i,j), i \neq j \\ i,j \in V_n}} U(i,j)(\sigma(i); \sigma(j)). \tag{1.1'}$$