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Nonanalytic Features of the First Order Phase Transition in the Ising Model

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Abstract. The absence of the analytic continuation for the free energy near the point of the first order phase transition in the d-dimensional Ising model is proved. It is shown that thermodynamic functions in the metastable phase do not have certain values and can be derived only with an uncertainty δ. The asymptotic expansion near the point of the phase transition yields the values of thermodynamic functions with the same uncertainty.

0. Introduction

The problem of existence of the analytic continuation for thermodynamic functions beyond the point of the first order phase transition is closely connected to the nature of metastable states. In the present work it will be shown that in the d-dimensional ($d \geqq 2$) ferromagnetic Ising model with nearest-neighbour interactions and for low temperatures there is no analytic continuation near the zero point of the magnetic field. Namely, let $F(E, h)$ be the free energy for the model where $E = 2 \cdot I/T$, $h = 2 \cdot H/T$, I is the energy of interaction, H is the magnetic field and T is the temperature. Then the following equality is the main result of this paper:

$$\lim_{h \to -0} \frac{1}{k!} \frac{\partial^k F}{\partial h^k} = (k!)^{\frac{1}{d-1}} (2(d-1)(E + C\xi))^{-\frac{kd}{d-1}},$$

where $k > E^d$, $C = C(d)$ depends only on the dimension and $|\xi| \leqq 1$ for sufficiently large E (see Sect. 3). The result will elucidate many features of the behaviour of the lattice gas in the metastable state.

A modification of the cluster expansion for the partition function will be used to obtain the main result. Two sets of terms with the same meaning will be defined. One of them can describe a wide class of lattice models, whereas the second class refers only to the Ising model with nearest-neighbour interactions. The question can be raised whether this double definition is necessary since only one model is considered. The fact is that the generalized partition function may refer both to the partition function of the model and to mathematical expressions which are not