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An Exactly Solvable Model of a Multidimensional Incommensurate Structure

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Abstract. The paper considers the class of Schrödinger multidimensional discrete operators with quasi-periodic unbounded potential for which essentially complete spectral analysis may be carried out. In the case of sufficiently high incommensurability of almost-periods, the spectrum of such operators is found to be pure point and simple, the eigenfunctions exponentially localized and the low frequency conductivity exponentially small. In the one-dimensional case, for any incommensurability, the spectrum does not contain the absolutely continuous component, while for small incommensurability the spectrum is singular continuous.

1. Introduction

The spectral properties of differential and finite difference operators with almost periodic coefficients raise at present considerable interest. The reason is both a very wide range of their potential applications and the variety of types of spectral behaviour that depends on the coefficients forms and the arithmetic properties of almost periods. In terms of the theory of disordered systems, almost periodic operators, which model in the one-body approximation the so-called incommensurate structures, are in an intermediate position between completely ordered systems, represented in the same approximation by equations with periodic coefficients and completely disordered systems for which the respective coefficients are random functions with sufficiently good mixing properties. Whereas in these two cases the structures and behaviours of basic physical quantities may be regarded in quite a number of interesting cases as clear enough (at least qualitatively or on the theoretical physics level of rigour [8]), understanding of the incommensurate system is now living through its formation (cf., e.g., refs. 1 and 13).

This paper, whose principal results were announced in ref. 11, studies the structure of the spectrum and estimates the low frequency conductivity of the incommensurate structure model described by a family of Hamiltonians $H_d(\omega)$