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Structure of Supermanifolds and Supersymmetry Transformations

Ugo Bruzzo and Roberto Cianci*

Istituto di Matematica, Università di Genova,
Via L. B. Alberti 4, I-16132 Genova, Italy

Abstract. After giving a global, constraint-free Lagrangian formulation of the $N = 1$ superspace supergravity in terms of super fibre bundles and differential forms over a supermanifold, we show that the concept of body manifold of a supermanifold provides a natural manner to reduce the theory to spacetime. This reduction, however, is not canonical, and the various ways in which it can be done give rise to transformations of the field variables which generalise the known invariances of the $N = 1$ spacetime supergravity under supersymmetry transformations and spacetime diffeomorphisms.

1. Introduction

The introduction of superspace [1] allows us to regard supergravity as a geometrical theory, contrary to spacetime supergravity, which is the theory of a spin-3/2 matter field interacting with a geometrical (gravitational) field. Superspace supergravity has been first formulated by Wess and Zumino [2]; as it stands, it is a purely local theory. The analogy with general relativity suggests the introduction of a manifold M, locally modelled on superspace. This leads to the concept of supermanifold, which in recent years has been the object of an intensive research [3–7], and seems to be the key for a global geometric formulation of supergravity.

In this framework, a difficulty immediately arises: how to connect the spacetime theory with the theory formulated on the supermanifold. Actually, under weak assumptions, a supermanifold M defines an ordinary manifold M_0 together with a well-behaved projection $\Phi: M \to M_0$. It is quite natural to identify M_0 with spacetime, but, since in general an immersion $\iota: M_0 \to M$ such that $\Phi \circ \iota = \mathrm{id}_{M_0}$ fails to exist, one is not able to pull back the theory onto M_0. This can be done with ease locally: given an open set $V \subset M$ with local coordinates (x^i, ξ^α)(x^i even, ξ^α odd), we may look at the points of V with coordinates $(x^i \in \mathbb{R}^4, 0)$ as the image of

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