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# Monopoles and Maps from $S^2$ to $S^2$; the Topology of the Configuration Space

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**Abstract.** The configuration space for the SU(2)-Yang-Mills-Higgs equations on $\mathbb{R}^3$ is shown to be homotopic to the space of smooth maps from $S^2$ to $S^2$. This configuration space indexes a family of twisted Dirac operators. The Dirac family is used to prove that the configuration space does not retract onto any subspace on which the SU(2)-Yang-Mills-Higgs functional is bounded.

## A1. Introduction

In [1], the author announced a theorem which stated that the SU(2) Yang-Mills equations on $\mathbb{R}^3$ in the Prasad Sommerfield limit have an infinite number of non-minimal (and gauge inequivalent) solutions in each path component of the configuration space (monopole sector). It was also asserted that solutions exist in each path component with arbitrarily large action. These assertions are proved in a forthcoming article [2] with techniques from the calculus of variations.

The calculus of variations can be used to find solutions to a differential equation if that equation is the Euler-Lagrange variational equation for a functional $f$ on a topological space $M$. If the pair $(f, M)$ are "nice" in a suitable sense, then certain topological properties of $M$ imply the existence of solutions to the differential equation. To make a concrete statement, one must study the functional $f$ and the topology of the space $M$.

The purpose of this article is to explore those topological properties of the Yang-Mills-Higgs configuration space which are relevant for the proof of the existence theorem in [1].

This exploration leads, among other places, to the topology of the family of Dirac operators indexed by this configuration space; here the characteristic classes of the family of Dirac operators are of specific interest. As outlined in Sect. 2, these cohomology classes lead to a proof that there exist solutions in each path component of the configuration space with arbitrarily large action.

The work here is based upon the preliminary topological investigations in Sect. 3 of [3]. Most of the terminology and notation in the present article is the same as in [3].

For the uninitiated, the SU(2) Yang-Mills-Higgs equations are partial differential equations on $\mathbb{R}^3$ for an unknown, $c=(A, \Phi)$. Here $A$ is a connection on

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